RSA 40 years later: A historical perspective

Paris May 30 2018

Jacques Stern
École normale supérieure

Summary

1. RSA before RSA
2. Did RSA prove secure enough?
3. Did RSA prove versatile enough?
4. Did RSA change our lives?

1976->1978 Only 2 years

644 IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. IT-22, NO. 6, NOVEMBER 1976

New Directions in Cryptography

Invited Paper

WHITFIELD DIFFIE AND MARTIN E. HELLMAN, MEMBER, IEEE

***Abstract*—Two kinds of contemporary developments in cryptography are examined. Widening applications of teleprocessing have given rise to a need for new types of cryptographic systems, which minimize the need for secure key distribution channels and supply the equivalent of a written signature. This paper suggests ways to solve these currently open problems. It also discusses how the theories of communication and computation are beginning to provide the tools to solve cryptographic problems of long standing.**

I. Introduction

WE STAND TODAY on the brink of a revolution in cryptography. The development of cheap digital hardware has freed it from the design limitations of me-

The best known cryptographic problem is that of privacy: preventing the unauthorized extraction of information from communications over an insecure channel. In order to use cryptography to insure privacy, however, it is currently necessary for the communicating parties to share a key which is known to no one else. This is done by sending the key in advance over some secure channel such as private courier or registered mail. A private conversation between two people with no prior acquaintance is a common occurrence in business, however, and it is unrealistic to expect initial business contacts to be postponed long enough for keys to be transmitted by some physical means. The cost and delay imposed by this key distribution problem is a major barrier to the transfer of business communications to large teleprocessing networks

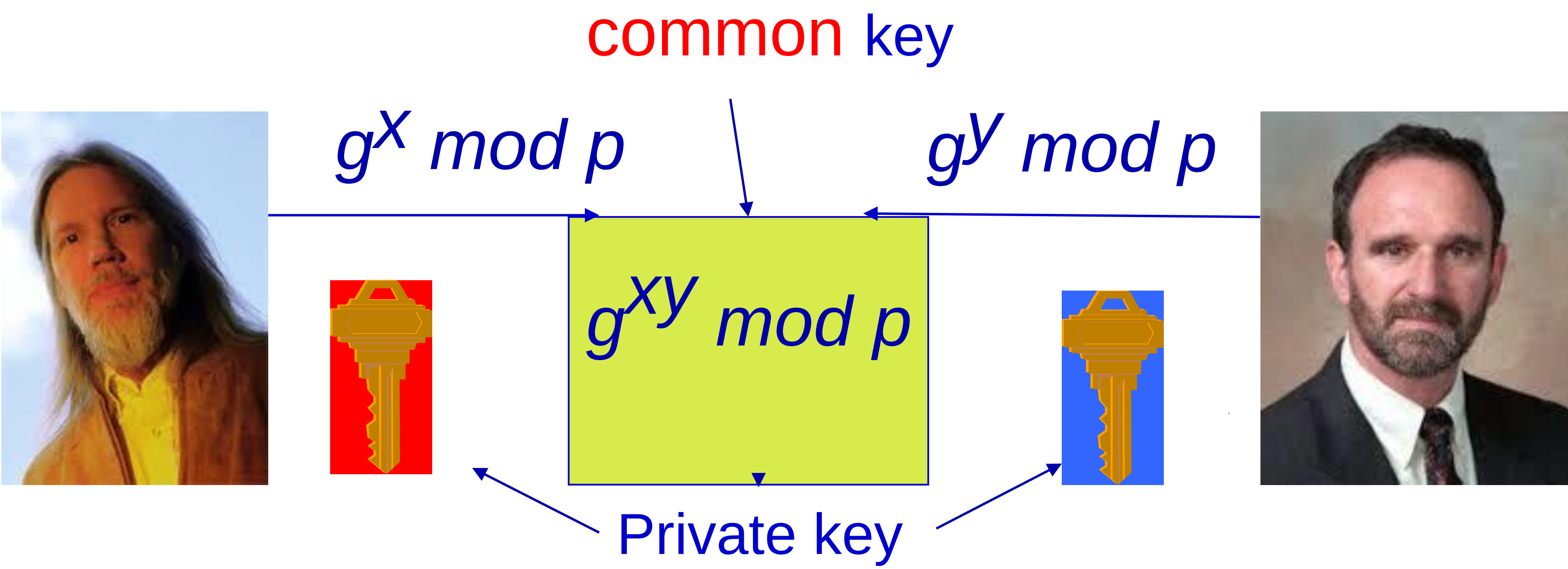
A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

R.L. Rivest, A. Shamir, and L. Adleman*

Abstract

An encryption method is presented with the novel property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences:

1976: Asymmetric Cryptography



- Whitfield Diffie and Martin Hellman 1976
- Secret key exchange

1976 ->1984: From DH to EG

ephemeral key

random *r*

$h=g^{x} \bmod p$

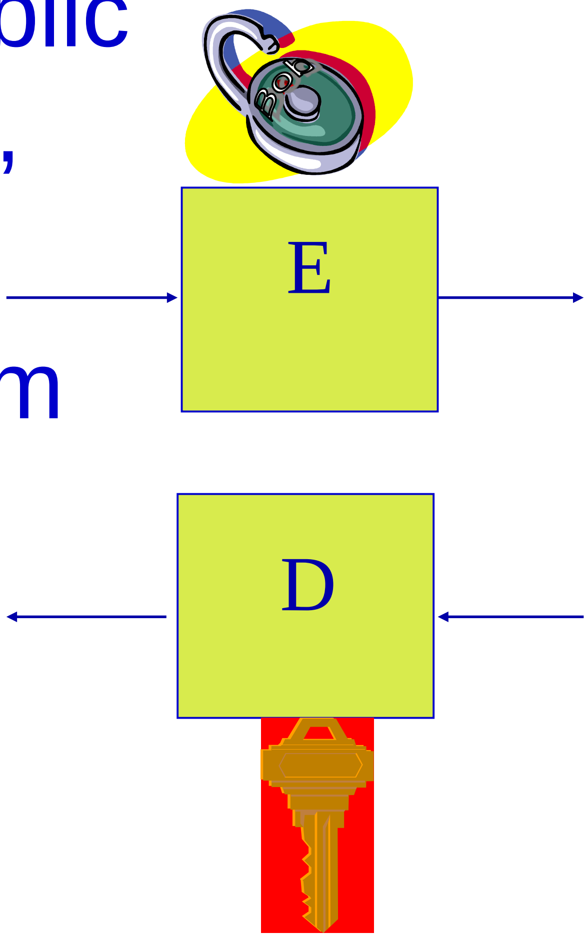
$g^{xr} \bmod p$

Private key

- El Gamal 1984
- Encrypt by producing $g^r \bmod p$ and using ephemeral key as mask for message : $m.h^r \bmod p$

1976->1978 From PKC to RSA

- 1976: Invention of PKC (Public Key Cryptography) by Diffie, Hellman
- 1978: The RSA cryptosystem and signature scheme by Rivest, Shamir, Adleman

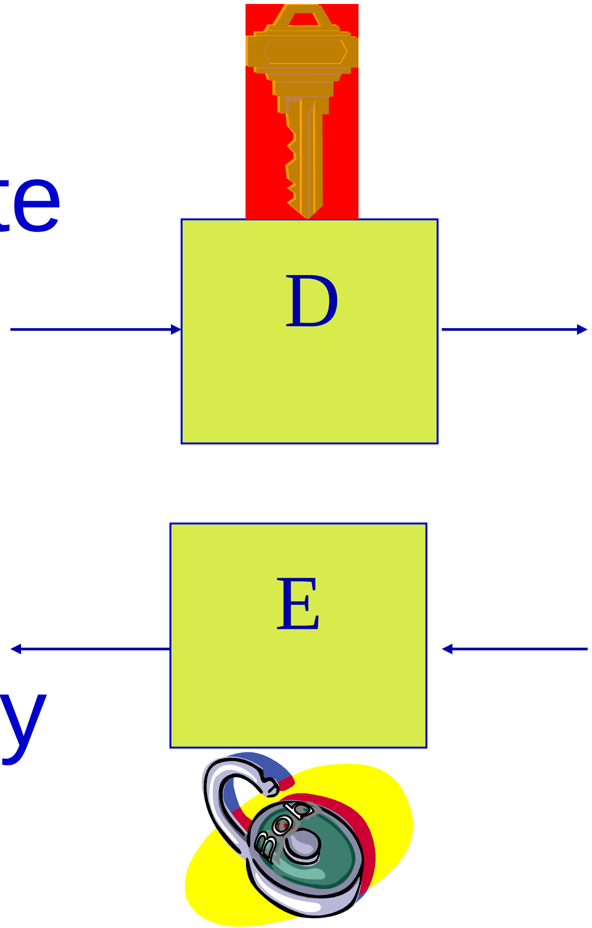


$$y=x^e \bmod n$$

$$n=pq \; p,q \; prime$$

RSA yields signatures

- Because E and D commute (known to DH)
- Apply D to message m to create signature
- Verify using public key only



1763 -> 1978: 215 years

74 (6)

THEOREMATA ARITHMETICA
NOVA METHODO DEMONSTRATA.

Auctore

L. EVLERO.

Praeter varias computandi operationes, quae vulgo in Arithmetica tradi ſolent, huiusque diſciplinae quaſi partem practicam conſtituunt, eiusdem pars Theoretica, quae in indaganda numerorum natura verſatur, non minus iam olim tractari eſt coepta, quemadmodum ex *Euclide* et *Diophanto* intelligere licet, vbi inſignes numerorum proprietates erutae reperiuntur ac demonſtratae. Quo magis autem deinceps numerorum indolem et affectiones Mathematici ſunt ſcrutati, multo plures eorum proprietates obſeruauerunt, vnde pulcherrima Theoremata numerorum naturam illuſtrantia deriuauere, quae partim demonſtrationibus ſunt munita, partim etiam nunc iis indigent, ſiue quod eae ab auctoribus non ſint inuentae, ſiue temporum iniuria deperditae: ex quo genere plurima paſſim occurrunt huiusmodi Theoremata numerica, quorum demonſtrationes adhuc deſiderantur; etiamſi eorum veritatem in dubium vocare non liceat. Atque hic inſigne diſcrimen, quod inter Theoremata arithmetica et geometrica intercedit, non parum mirari debemus, quod vix vlla propoſitio geometrica proferri poſſit, quam non ſit in promtu, ſiue veram, ſiue falſam, oſtendere, dum

- Novi Commentarii Academiae Scientarum Petropolitanae 8, 1763, 74-104

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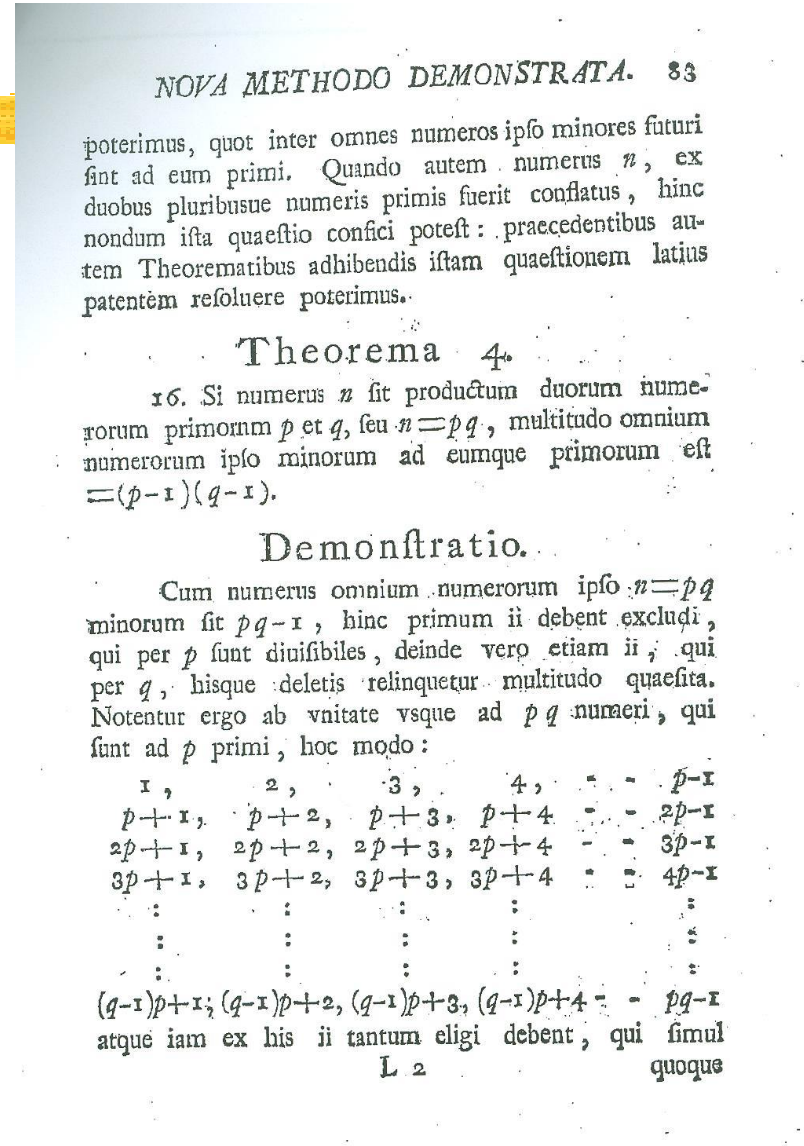
Abstract

An encryption method is presented with the novel property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences:

Euler 1763

- Page 83:the numbers of $< n$ integers prime to n is equal to $\phi(n) = (p-1)(q-1)$



poterimus, quot inter omnes numeros ipſo minores futuri ſint ad eum primi. Quando autem numerus n, ex duobus pluribusue numeris primis fuerit conflatus, hinc nondum iſta quaeſtio confici poteſt: praecedentibus autem Theorematibus adhibendis iſtam quaeſtionem latius patentem reſoluere poterimus.

Theorema 4.

16. Si numerus n ſit productum duorum numerorum primorum p et q, ſeu $n = pq$, multitudo omnium numerorum ipſo minorum ad eumque primorum eſt $= (p-1)(q-1)$.

Demonſtratio.

Cum numerus omnium numerorum ipſo $n = pq$ minorum ſit $pq-1$, hinc primum ii debent excludi, qui per p ſunt diuiſibiles, deinde vero etiam ii, qui per q, hisque deletis relinquetur multitudo quaeſita. Notentur ergo ab vnitate vsque ad pq numeri, qui ſunt ad p primi, hoc modo:

$1,$	$2,$	$3,$	$4,$	- -	$p-1$
$p+1,$	$p+2,$	$p+3,$	$p+4$	- -	$2p-1$
$2p+1,$	$2p+2,$	$2p+3,$	$2p+4$	- -	$3p-1$
$3p+1,$	$3p+2,$	$3p+3,$	$3p+4$	- -	$4p-1$
⋮	⋮	⋮	⋮		⋮
$(q-1)p+1,$	$(q-1)p+2,$	$(q-1)p+3,$	$(q-1)p+4$	- -	$pq-1$

atque iam ex his ii tantum eligi debent, qui ſimul

L 2 quoque

Looks like the basis for RSA

Coroll. 2.

49. Contra autem iam ſupra vidimus productum ex duobus pluribusue reſiduis in claſſe reſiduorum reperiri. Vnde ſequitur ex vno non-reſiduo et quotcunque reſiduis in claſſe non-reſiduorum occurrere debere.

Scholion.

50. Vis huius demonſtrationis iſto nititur fundamento, quod ſi inter reſidua occurrant partes 1, a, b, c, d, etc. ad diuiſorem primae, atque α fuerit etiam pars ad diuiſorem prima in his reſiduis non contenta, tum producta omnia αa, αb, αc, αd, etc. non ſolum in reſiduis non occurrere, quod quidem perfecte eſt demonſtratum, ſed etiam ea eſſe partes ad diuiſorem N primas, omnesque inter ſe diuerſas; ſeu ſi ea per N, actu diuidantur, relinqui reſidua diuerſa. Illud quidem per ſe eſt perſpicuum; cum enim tam α, quam a, b, c, d, etc. ſint numeri ad N primi, etiam eorum producta ad N prima ſint neceſſe eſt. Quod autem producta αa, αb, αc, αc, etc. ſint omnia ad N relata inter ſe diuerſa, intelligitur, quod ſi verbi gratia duo αa et αb per N diuiſa paria darent reſidua, eorum differentia $\alpha b - \alpha a = \alpha(b-a)$ per N eſſet diuiſibilis, ideoque et $b-a$; id quod hypotheſi, quod a et b ſint diverſae partes ad N primae, repugnat.

Theorema 10.

51. Exponens minimae poteſtatis x^{ν}, quae per numerum N ad x primum diuiſa vnitatem relinquit, vel

vel eſt aequalis numero partium ad N primarum, vel huius numeri ſemiſſis, aliaue eius pars aliquota.

Demonſtratio.

Sit n numerus partium ad N primarum, quarum cum ν conſtituant reſidua, erit numerus non-reſiduorum $= n-\nu$. Vidimus autem hunc numerum eſſe vel $=0$, vel $=\nu$, vel $=2\nu$, vel alii cuipiam multiplo exponentis ν. Sit ergo $n-\nu=(m-1)\nu$, ita vt m denotet vel vnitatem, vel alium quemuis numerum integrum, atque hinc obtinebimus $n=m\nu$ et $\nu=\frac{n}{m}$: vnde patet exponentem minimae poteſtatis ipſius x, quae per N diuiſa vnitatem relinquit, eſſe vel $=n$, ſi $m=1$, vel $=\frac{n}{2}$, ſi $m=2$, vel in genere eſſe partem quampiam aliquotam numeri n, qui exprimit multitudinem partium ad diuiſorem N primarum. Q. E. D.

Coroll. 1.

52. Si x^{ν} fuerit minima poteſtas, quae per numerum N ad x primum diuiſa vnitatem relinquit, ſequentes poteſtates idem reſiduum relinquentes ſunt $x^{2\nu}$, $x^{3\nu}$, $x^{4\nu}$, $x^{5\nu}$, etc. neque praetera vllae aliae dantur, quae per N diuiſae vnitatem relinquant.

Coroll. 2.

53. Exponens ergo huius poteſtatis minimae ſemper cum numero partium ad diuiſorem N primarum ita connectitur, vt ſit vel illi ipſi, vel cuipiam eius parti aliquotae, aequalis.

Scholion.

1978: what is the future of RSA

- Will RSA prove secure enough? Or shall we give it up?
- Will RSA prove versatile enough? Or shall we need alternatives?
- Will RSA change our lives?

First ten years: textbook attacks

- Small message space attack: exhaustively compute $E_K(m_i)$ until correct message is found
- Broadcast attack : obtain encryption of an identical message under various public keys (Hastad)
- Solve equations by chinese remaindering

$$x^3 = c \bmod N_1$$

.....

$$x^3 = c \bmod N_3$$

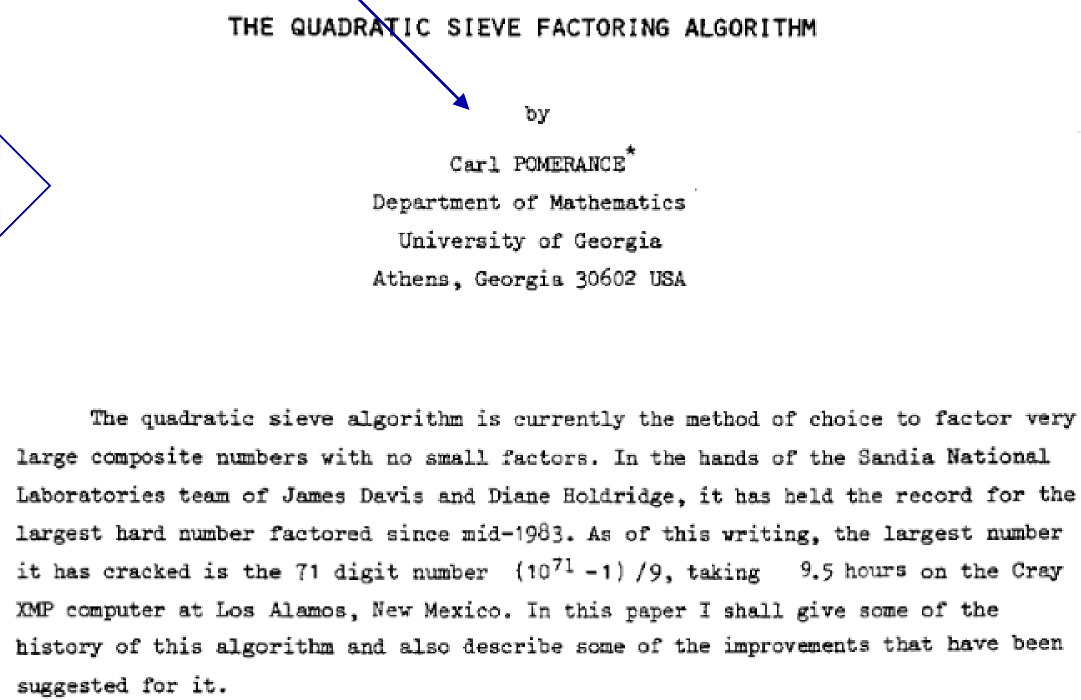
First 10 years: factoring

- M. Kraitchik, Recherches sur a Théorie des Nombres, Tome 2, Factorisation, Gauthier-Villars, Paris, 1929.
- Factor and combine congruences to yield $x^2 = y^2 \bmod N$

Pomerance, Eurocrypt 84, Paris


RECHERCHES

THÉORIE DES NOMBRES


THE QUADRATIC SIEVE FACTORING ALGORITHM

by

Carl POMERANCE*

Department of Mathematics

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The quadratic sieve algorithm is currently the method of choice to factor very large composite numbers with no small factors. In the hands of the Sandia National Laboratories team of James Davis and Diane Holdridge, it has held the record for the largest hard number factored since mid-1983. As of this writing, the largest number it has cracked is the 71 digit number $(10^{71}-1)/9$, taking 9.5 hours on the Cray XMP computer at Los Alamos, New Mexico. In this paper I shall give some of the history of this algorithm and also describe some of the improvements that have been suggested for it.

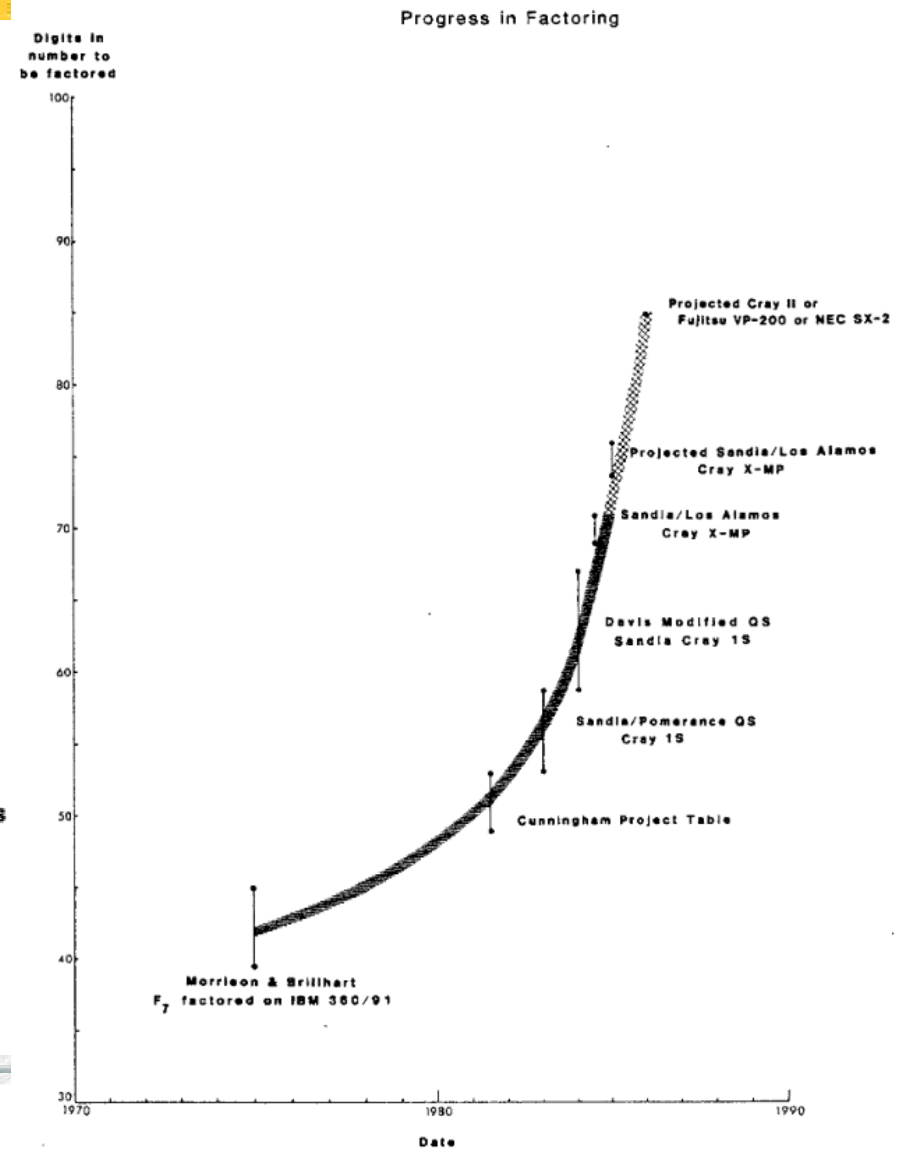
First 10 years: factoring

- Also from Eurocrypt 84

Status Report on Factoring
(At the Sandia National Laboratories)*

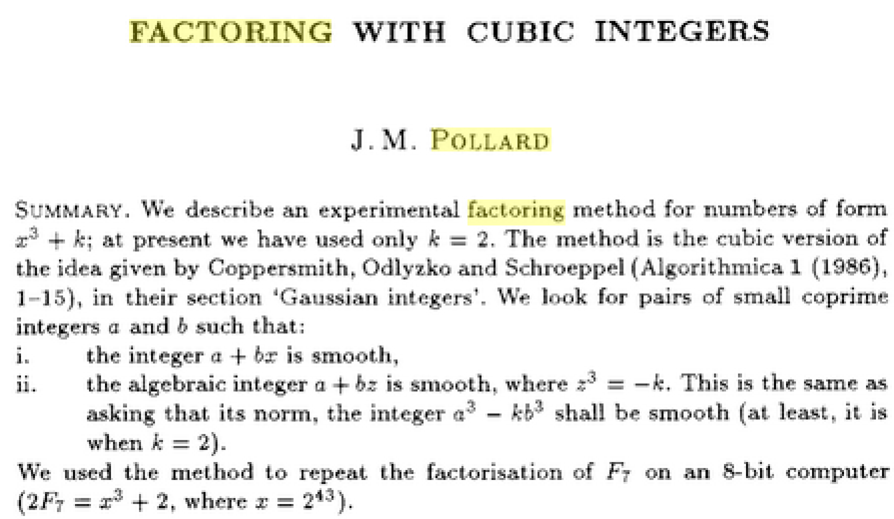
James A. Davis, Diane B. Holdridge and Gustavus J. Simmons

Sandia National Laboratories
Albuquerque, New Mexico 87185



1988: a breaking point

- A 6 pages manuscript by John Pollard circulated in August 1988.
- Published a few years later in : The Development of the Number Field Sieve Lenstra, Arjen K., Lenstra, Hendrik W.Jr. (Eds.)


FACTORING WITH CUBIC INTEGERS

J. M. POLLARD

SUMMARY. We describe an experimental factoring method for numbers of form $x^3 + k$; at present we have used only $k = 2$. The method is the cubic version of the idea given by Coppersmith, Odlyzko and Schroeppel (Algorithmica **1** (1986), 1–15), in their section 'Gaussian integers'. We look for pairs of small coprime integers a and b such that:

i. the integer $a + bx$ is smooth,

ii. the algebraic integer $a + bz$ is smooth, where $z^3 = -k$. This is the same as asking that its norm, the integer $a^3 - kb^3$ shall be smooth (at least, it is when $k = 2$).

We used the method to repeat the factorisation of F_7 on an 8-bit computer ($2F_7 = x^3 + 2$, where $x = 2^{43}$).

Second 10 years : factoring

```
RSA-100
Factors: 40094690950920881030683735292761468389214899724061 * 37975227936943673922808872755445627854565536638199
Date: April 1, 1991
Method: ppmpqs
Time: Approx. 7 MIP-Years
Name: Mark Manasse, Arjen K. Lenstra
Email: msm@src.dec.com, lenstra@flash.bellcore.com
Recd: April 1, 1991
-----------------------------------------------------------------

 We are happy to announce that

 RSA-129 = 114381625757888867669235779976146612010218296721242362562561842\
           35706935245733897830597123563958705058989075147599290026879543541
         = 3490529510847650949147849619903898133417764638493387843990820577 *
           32769132993266709549961988190834461413177642967992942539798288533

 The encoded message published was

 96869613754622061477140922254355882905759991124574319874695120930816298225145708356931476622883989628013391990551829945157815154

 This number came from an RSA encryption of the `secret' message using the
 public exponent 9007. When decrypted with he `secret' exponent

 106698614368578024442868771328920154780709906633937862801226224496631\
 063125911774470873340168597462306553968544513277109053606095

 this becomes

 200805001301070903002315180419000118050019172105011309190800151919090\
 618010705

 Using the decoding scheme 01=A, 02=B, ..., 26=Z, and 00 a space between
 words, the decoded message reads

 THE MAGIC WORDS ARE SQUEAMISH OSSIFRAGE
```

- April 1, 1991 RSA 110
- April 26, 1994 RSA 129 original RSA 100 $ challenge
- April 10, 1996 RSA 130 using GNFS, 8 years after Pollard's manuscript

1993: Engineering (PKCS#1 v 1.5)

00	02	Random pad	00	Data block

Used in SSL v3.0

00	02	Random pad	00	03	00	premastersecret

46 bytes (premastersecret)

Algebraic attacks revisited

- Tool : method to solve a low degree polynomial equation $P(x)=0 \bmod N$, when suitable approximation of a root is given (Coppersmith 94)
- Applies to factoring when partial information is known
- Also applies to small message space with random padding : Randomness should not be too small

1994 : the quantum threat

- Proceedings of the 35th FOCS, Santa Fe, NM, Nov. 20--22, 1994
- SIAM J.Sci.Statist.Comput. 26 (1997) 1484

Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor†

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

1998: The oracle threat

Chosen Ciphertext Attacks Against Protocols Based on the RSA Encryption Standard PKCS #1

Daniel Bleichenbacher

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700 Mountain Ave.
Murray Hill, NJ 07974
E-mail: bleichen@research.bell-labs.com

Abstract. This paper introduces a new adaptive chosen ciphertext attack against certain protocols based on RSA. We show that an RSA private-key operation can be performed if the attacker has access to an oracle that, for any chosen ciphertext, returns only one bit telling whether the ciphertext corresponds to some unknown block of data encrypted using PKCS #1. An example of a protocol susceptible to our attack is SSL V.3.0.

- To decrypt c, submit ciphertext cs^e
- Usually not PKCS#1 compliant
- if accepted reveal 7 bit of info
- Repeat cleverly
- cleartext is recovered after a few thousand calls
- plausible in SSL setting

Third 10 years : factoring

- August 22, 1999, 512 bits!

```
RSA-155
Factors:
102639592829741105772054196573991675900716567808038066803341933521790711307779
*
106603488380168454820927220360012878679207958575989291522270608237193062808643
Date:     August 22, 1999
Method:   the General Number Field Sieve,
          with a polynomial selection method of Brian Murphy
          and Peter L. Montgomery,
          with lattice sieving (71%) and with line sieving (29%),
          and with Peter L. Montgomery's blocked Lanczos and
          square root algorithms;
```

- December 3, 2003

RSA-576 has 174 decimal digits (576 bits), and was factored on December 3, 2003 by J. Franke and T. Kleinjung from the University of Bonn. factorization.

The value and factorization are as follows:

```
RSA-576 = 188198812920607963838697239461650439807163563379417382700763356422988859715234665485319
          060606504743045317388011303396716199692321205734031879550656996221305168759307650257059
```

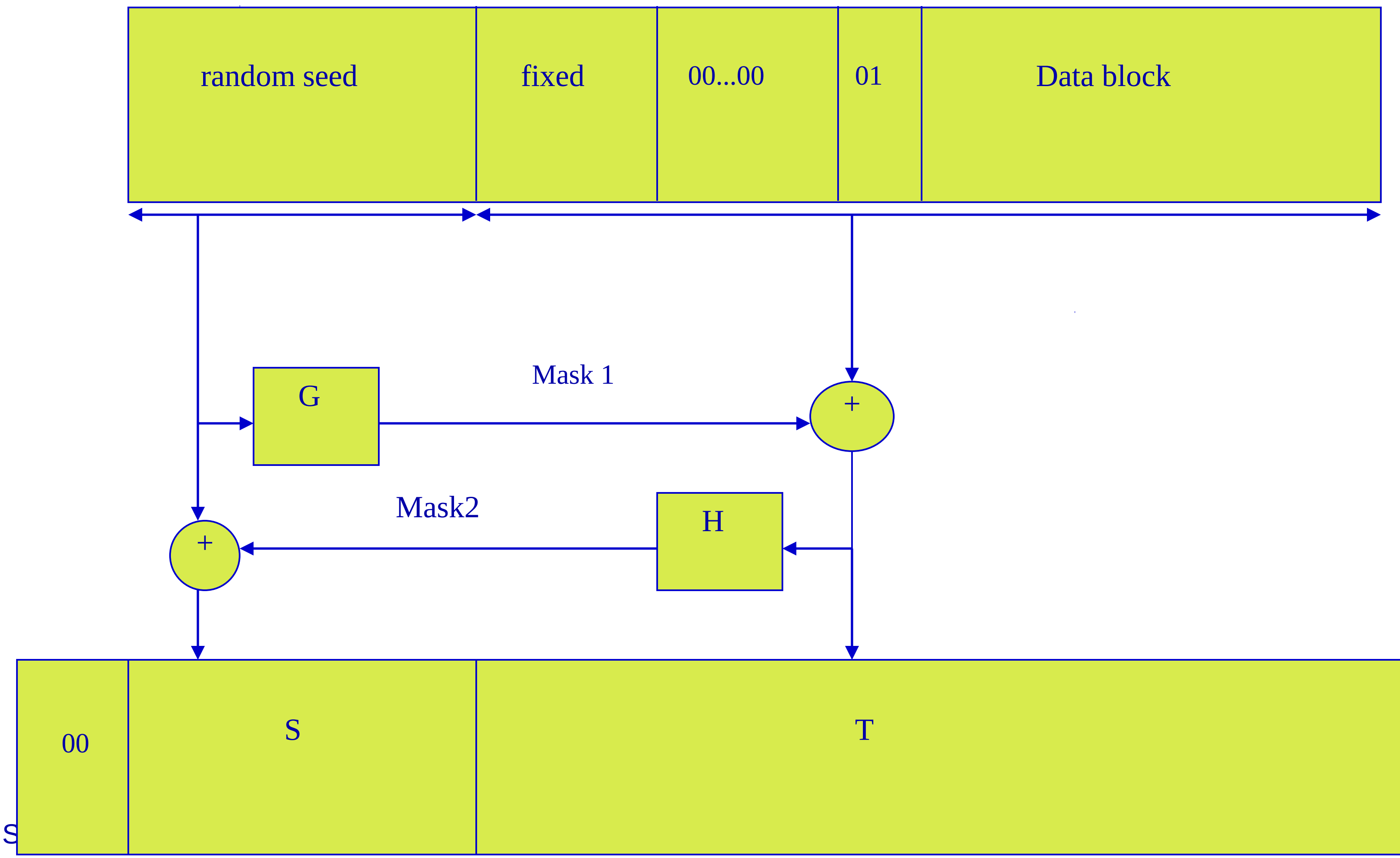
```
RSA-576 = 398075086424064937397125500550386491199064362342526708406385189575946388957261768583317
        × 472772146107435302536223071973048224632914695302097116459852171130520711256363590397527
```

The factorization was found using the general number field sieve algorithm.

Third 10 years : provable security

- Provide a mathematical proof that formatted RSA is “as secure” as full-size “raw” RSA
- Hash functions are treated as purely random
- Adversary is extensively allowed to query decryption of related (but distinct from target) ciphertexts [CCA attack]
- Still unable to get one bit of information on target

1994: OAEP (Bellare Rogaway)



2001: The OAEP saga

- OAEP Believed to withstand CCA attacks,
- Paper by Shoup showing proof invalid
- Repaired by FOPS same year

OAEP Reconsidered*

Victor Shoup

IBM Zurich Research Lab, Säumerstr. 4, 8803 Rüschlikon, Switzerland

sho@zurich.ibm.com

September 18, 2001

Abstract

The OAEP encryption scheme was introduced by Bellare and Rogaway at Eurocrypt '94. It converts any trapdoor permutation scheme into a public-key encryption scheme. OAEP is widely believed to provide resistance against adaptive chosen ciphertext attack. The main justification for this belief is a supposed proof of security in the random oracle model, assuming the underlying trapdoor permutation scheme is one way.

This paper shows conclusively that this justification is invalid. First, it observes

RSA–OAEP is Secure under the RSA Assumption*

Eiichiro Fujisaki and **Tatsuaki Okamoto**

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URL: http://www.di.ens.fr/users/{pointche,stern}

Fourth 10 years : factoring

- December 12, 2009 « halfway » to 1024 bits !
- Later: smaller sizes

RSA-210 [*]	210	696		September 26, 2013[8]	Ryan Propper
RSA-704 [*]	212	704	$30,000 USD	July 2, 2012	Shi Bai, Emmanuel Thomé and Paul Zimmermann
RSA-220 [*]	220	729		May 13, 2016	S. Bai, P. Gaudry, A. Kruppa, E. Thomé and P. Zimmermann
RSA-230	230	762			
RSA-232	232	768			
RSA-768 [*]	232	768	$50,000 USD	December 12, 2009	Thorsten Kleinjung *et al.*

Quantum factoring

- Still not competing

Table 5: *Quantum factorization records*

Number	# of factors	# of qubits needed	Algorithm	Year implemented	Implemented without prior knowledge of solution
15	2	8	Shor	2001 [2]	✗
	2	8	Shor	2007 [3]	✗
	2	8	Shor	2007 [3]	✗
	2	8	Shor	2009 [5]	✗
	2	8	Shor	2012 [6]	✗
21	2	10	Shor	2012 [7]	✗
143	2	4	minimization	2012 [1]	✓
56153	2	4	minimization	2012 [1]	✓
291311	2	6	minimization	not yet	✓
175	3	3	minimization	not yet	✓

High-fidelity adiabatic quantum computation using the intrinsic Hamiltonian of a spin system: Application to the experimental factorization of 291311

Zhaokai Li, Nikesh S. Dattani, Xi Chen, Xiaomei Liu, Hengyan Wang, Richard Tanburn, Hongwei Chen, Xinhua Peng, Jiangfeng Du

(Submitted on 25 Jun 2017)

In previous implementations of adiabatic quantum algorithms using spin systems, the average Hamiltonian method with Trotter's formula was conventionally adopted to generate an effective instantaneous Hamiltonian that simulates an adiabatic passage. However, this approach had issues with the precision of the effective Hamiltonian and with the adiabaticity of the evolution. In order to address these, we here propose and experimentally demonstrate a novel scheme for adiabatic quantum computation by using the intrinsic Hamiltonian of a realistic spin system to represent the problem Hamiltonian while adiabatically driving the system by an extrinsic Hamiltonian directly induced by electromagnetic pulses. In comparison to the conventional method, we observed two advantages of our approach: improved ease of implementation and higher fidelity. As a showcase example of our approach, we experimentally factor 291311, which is larger than any other quantum factorization known.

Subjects: **Quantum Physics (quant-ph)**
Cite as: **arXiv:1706.08061 [quant-ph]**
(or **arXiv:1706.08061v1 [quant-ph]** for this version)

Submission history

From: Jiangfeng Du [view email]
[v1] Sun, 25 Jun 2017 08:53:02 GMT (1276kb,D)

Beyond provable security

- Verify cryptographic proofs formally
- Active research
- Many success with proof assistants

Beyond Provable Security
Verifiable IND-CCA Security of OAEP

Gilles Barthe[1], Benjamin Grégoire[2],
Yassine Lakhnech[3], and Santiago Zanella Béguelin[1]

[1] IMDEA Software
[2] INRIA Sophia Antipolis-Méditerranée
[3] Université Grenoble 1, CNRS, Verimag

Abstract. OAEP is a widely used public-key encryption scheme based on trapdoor permutations. Its security proof has been scrutinized and amended repeatedly. Fifteen years after the introduction of OAEP, we present a machine-checked proof of its security against adaptive chosen-ciphertext attacks under the assumption that the underlying permutation is partial-domain one-way. The proof can be independently verified by running a small and trustworthy proof checker and fixes minor glitches that have subsisted in published proofs. We provide an overview of the proof, highlight the differences with earlier works, and explain in some detail a crucial step in the reduction: the elimination of indirect queries made by the adversary to random oracles via the decryption oracle. We also provide—within the limits of a conference paper—a broader perspective on independently verifiable security proofs.

Back in 1978: RSA versatile?

- CANNOT provide short keys
- CANNOT allow to use email address as PK
- CANNOT allow to perform Crypto-computing

- Fostered 40 years of research on alternatives

1985: Shorter keys via EC

- Shorter keys due to less efficient attacks
- Miller
- Koblitz

Use of Elliptic Curves in Cryptography

Victor S. Miller

Exploratory Computer Science, IBM Research, P.O. Box 218, Yorktown Heights, NY 10598

ABSTRACT

We discuss the use of elliptic curves in cryptography. In particular, we propose an analogue of the Diffie-Hellmann key exchange protocol which appears to be immune from attacks of the style of Western, Miller, and Adleman. With the current bounds for infeasible attack, it appears to be about 20% faster than the Diffie-Hellmann scheme over GF(p). As computational power grows, this disparity should get rapidly bigger.

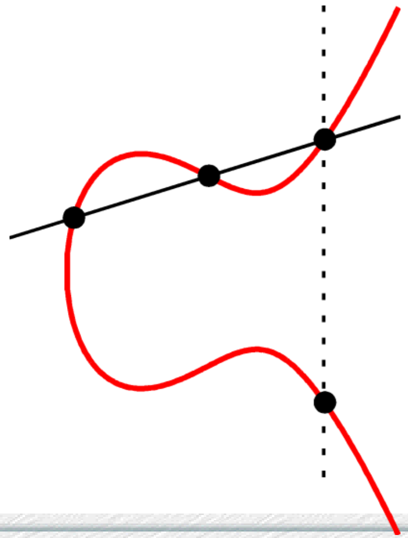
MATHEMATICS OF COMPUTATION
VOLUME 48, NUMBER 177
JANUARY 1987, PAGES 203-209

Elliptic Curve Cryptosystems

By Neal Koblitz

This paper is dedicated to Daniel Shanks on the occasion of his seventieth birthday

Abstract. We discuss analogs based on elliptic curves over finite fields of public key cryptosystems which use the multiplicative group of a finite field. These elliptic curve cryptosystems may be more secure, because the analog of the discrete logarithm problem on elliptic curves is likely to be harder than the classical discrete logarithm problem, especially over $GF(2^n)$. We discuss the question of primitive points on an elliptic curve modulo p, and give a theorem on nonsmoothness of the order of the cyclic subgroup generated by a global point.



1984: ID based

- PK related to ID
- Generated by Tusted third Party
- Proposed for signatures

IDENTITY-BASED CRYPTOSYSTEMS AND SIGNATURE SCHEMES

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THE IDEA

In this paper we introduce a novel type of cryptographic scheme, which enables any pair of users to communicate securely and to verify each other's signatures without exchanging private or public keys, without keeping key directories, and without using the services of a third party. The scheme assumes the existence of trusted key generation cen-

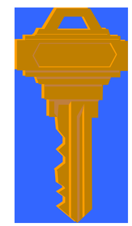
2000: Tripartite DH

common key

g^x

$e(g,g)^{xyz}$

g^y



g^z

A One Round Protocol for Tripartite Diffie–Hellman

Antoine Joux

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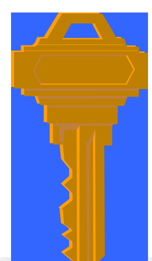
Abstract. In this paper, we propose a three participants variation of the Diffie–Hellman protocol. This variation is based on the Weil and Tate pairings on elliptic curves, which were first used in cryptography as cryptanalytic tools for reducing the discrete logarithm problem on some elliptic curves to the discrete logarithm problem in a finite field.



1940 -> 2000 Pairings

- Introduced by Weil 1940

ALGÈBRE. — *Sur les fonctions algébriques à corps de constantes fini.* Note (¹) de M. **André Weil**, présentée par M. Élie Cartan.

Je vais résumer dans cette Note la solution des principaux problèmes de la théorie des fonctions algébriques à corps de constantes fini ; on sait que celle-ci a fait l'objet de nombreux travaux, et plus particulièrement, dans les dernières années, de ceux de Hasse et de ses élèves ; comme ils l'ont entrevu, la théorie des correspondances donne la clef de ces problèmes ; mais la théorie algébrique des correspondances, qui est due à Severi, n'y suffit point, et il faut étendre à ces fonctions la théorie transcendante de Hurwitz.

- Used in Crypto to spot “weak” elliptic curves where DLP is easier
- Reversed by Joux 2000

2000 -> 2001: few months

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Compared to 6 years for DH -> EG !

Identity-Based Encryption from the Weil Pairing

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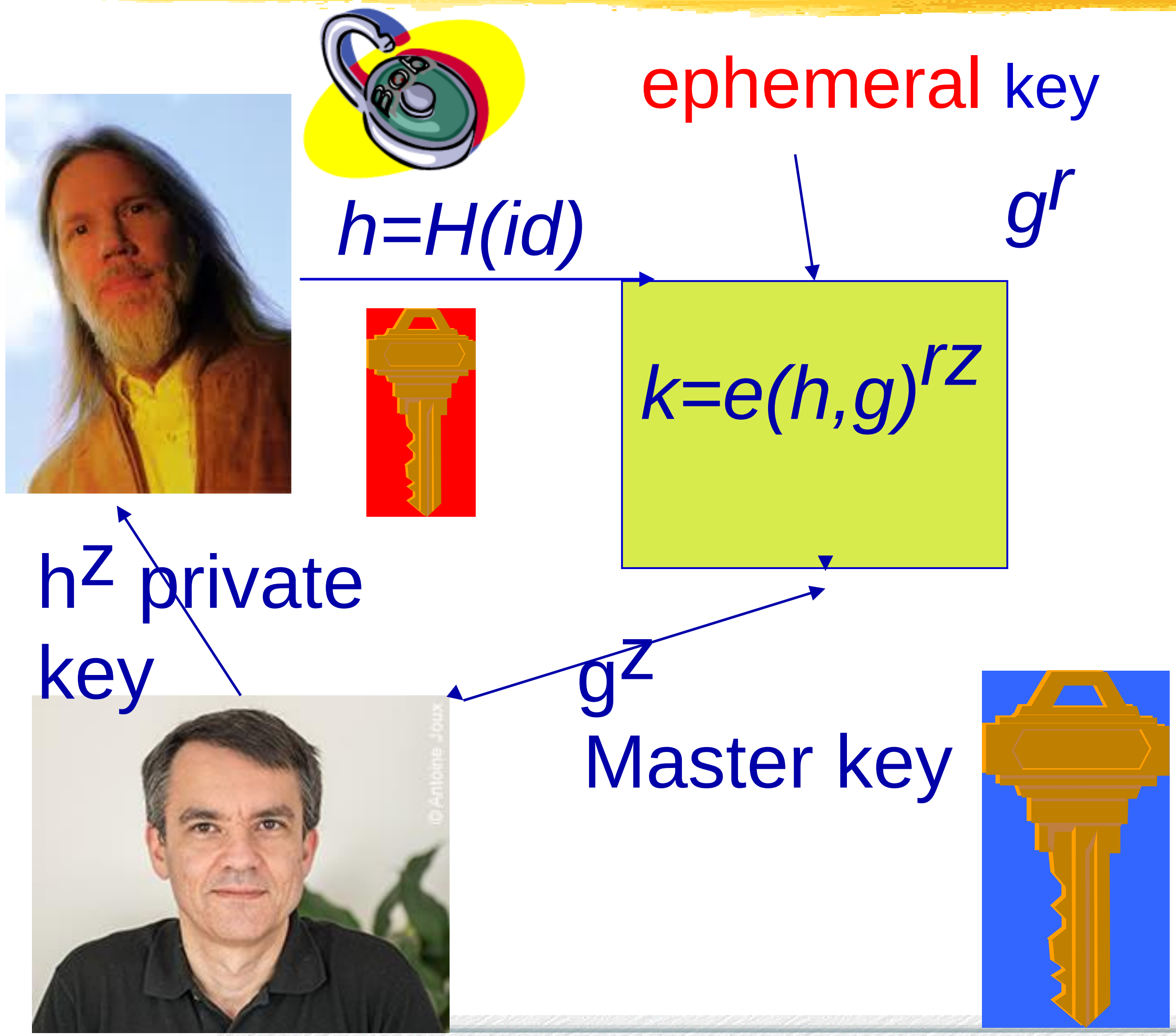
Matthew Franklin† franklin@cs.ucdavis.edu

Abstract

We propose a fully functional identity-based encryption scheme (IBE). The scheme has chosen ciphertext security in the random oracle model assuming a variant of the computational Diffie-Hellman problem. Our system is based on bilinear maps between groups. The Weil pairing on elliptic curves is an example of such a map. We give precise definitions for secure identity based encryption schemes and give several applications for such systems.

From TDH to ID-based

random r



- Encypts m as

  $(g^r, m.k)$

1978 -> 2009 : HE

ON DATA BANKS AND PRIVACY HOMOMORPHISMS

Ronald L. Rivest
Len Adleman
Michael L. Dertouzos

Massachusetts Institute of Technology
Cambridge, Massachusetts

I. INTRODUCTION

Encryption is a well-known technique for preserving the privacy of sensitive information. One of the basic, apparently inherent, limitations of this technique is that an information system working with encrypted data can at most store or retrieve the data for the user; any more complicated operations seem to require that the data be decrypted before being operated on.

A FULLY HOMOMORPHIC ENCRYPTION SCHEME

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF COMPUTER SCIENCE
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Craig Gentry
September 2009

Homomorphic encryption

- Many schemes have “somewhat homomorphic” properties
- Based on encrypting with noise and decrypting with trapdoor
- Too many operations on encrypted data does not allow to recover error

Bootstrapping

- Breakthrough by Gentry 09 boostrapping technique

Fully Homomorphic Encryption Using Ideal Lattices

Craig Gentry
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ABSTRACT

We propose a fully homomorphic encryption scheme – i.e., a scheme that allows one to evaluate circuits over encrypted data without being able to decrypt. Our solution comes in three steps. First, we provide a general result – that, to construct an encryption scheme that permits evaluation of *arbitrary circuits*, it suffices to construct an encryption scheme that can evaluate (slightly augmented versions of) its *own decryption circuit*; we call a scheme that can evaluate its (augmented) decryption circuit *bootstrappable*.

duced by Rivest, Adleman and Dertouzos [54] shortly after the invention of RSA by Rivest, Adleman and Shamir [55]. Basic RSA is a multiplicatively homomorphic encryption scheme – i.e., given RSA public key pk = (N, e) and ciphertexts $\{\psi_i \leftarrow \pi_i^e \bmod N\}$, one can efficiently compute $\prod_i \psi_i = (\prod_i \pi_i)^e \bmod N$, a ciphertext that encrypts the product of the original plaintexts. Rivest et al. [54] asked a natural question: What can one do with an encryption scheme that is *fully* homomorphic: a scheme $\mathcal{E}$ with an efficient algorithm $\mathsf{Evaluate}_{\mathcal{E}}$ that, for any valid public key pk, *any* circuit C (not just a circuit consisting of multiplication

- Used ideal lattices in Z[X]/f

2010-11 : Variants/implementations

Fully Homomorphic Encryption with Relatively Small Key and Ciphertext Sizes

Nigel P. Smart[1] and Frederik Vercauteren[2]

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Abstract. We present a fully homomorphic encryption scheme which has both relatively small key and ciphertext size. Our construction fol-

PKC09 EC 11

EC10

Implementing Gentry's Fully-Homomorphic Encryption Scheme

Craig Gentry⋆ and Shai Halevi⋆

IBM Research

Abstract. We describe a working implementation of a variant of Gentry's fully homomorphic encryption scheme (STOC 2009), similar to the variant used in an earlier implementation effort by Smart and Vercauteren (PKC 2010). Smart and Vercauteren implemented the underlying "somewhat homomorphic" scheme, but were not able to implement the bootstrapping functionality that is needed to get the complete scheme to work. We show a number of optimizations that allow us to implement all aspects of the scheme, including the bootstrapping functionality.

Fully Homomorphic Encryption over the Integers

Marten van Dijk[1], Craig Gentry[2], Shai Halevi[2], and Vinod Vaikuntanathan[2]

[1] MIT CSAIL
[2] IBM Research

Abstract. We construct a simple fully homomorphic encryption scheme, using only elementary modular arithmetic. We use Gentry's technique to construct a fully homomorphic scheme from a "bootstrappable" somewhat homomorphic scheme. However, instead of using ideal lattices over a polynomial ring, our bootstrappable encryption scheme merely uses addition and multiplication over the integers. The main appeal of our scheme is the conceptual simplicity.

HE over the integers

- Simpler construction
- Security based on the "approximate GCD" problem [*find an integer p from approximations of several multiples of p*
- Seems familiar to cryptanalysts …

The two faces of lattices

The Two Faces of Lattices in Cryptology

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Abstract. Lattices are regular arrangements of points in n-dimensional space, whose study appeared in the 19th century in both number theory and crystallography. Since the appearance of the celebrated Lenstra-Lenstra-Lovász lattice basis reduction algorithm twenty years ago, lattices have had surprising applications in cryptology. Until recently, the applications of lattices to cryptology were only negative, as lattices were used to break various cryptographic schemes. Paradoxically, several positive cryptographic applications of lattices have emerged in the past five years: there now exist public-key cryptosystems based on the hardness of lattice problems, and lattices play a crucial rôle in a few security proofs. We survey the main examples of the two faces of lattices in cryptology.

A method to break the approximate gcd problem (using orthogonal lattices see NS 2001)

A method to achieve HE

Alternatives in 2018

- Practicality is improving
- Balance between design and cryptanalysis not always clear
- Additional research needed

Cryptanalysis takes time

– 2001

FLASH, a fast multivariate signature algorithn
http://www.minrank.org/flash/

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Abstract. This article describes the particular parameter choice and implementation details of one of the rare published, but not broken signature schemes, that allow signatures to be computed and checked by a low-cost smart card. The security is controversial, since we have no proof of security, but the best known attacks require more than 2^{80} computations. We called FLASH our algorithm and we also proposed SFLASH, a version that has a smaller public key and faster verification though one should be even more careful about it's security.
FLASH and SFLASH have been accepted as submissions to NESSIE (New European Schemes for Signatures, Integrity, and Encryption), a project within the Information Societies Technology (IST) Programme of the European Commission.

2007

Practical Cryptanalysis of SFLASH

Vivien Dubois[1], Pierre-Alain Fouque[1], Adi Shamir[1,2], and Jacques Stern[1]

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Abstract. In this paper, we present a practical attack on the signature scheme SFLASH proposed by Patarin, Goubin and Courtois in 2001 following a design they had introduced in 1998. The attack only needs the public key and requires about one second to forge a signature for any message, after a one-time computation of several minutes. It can be applied to both SFLASH^{v2} which was accepted by NESSIE, as well as to SFLASH^{v3} which is a higher security version.

Did RSA change our lives?

- As a community YES
- Gathered smart researches
- Fostered progress in crypto
- Also in related fields : quantum, DPA, formal security …

Did RSA change our lives?

- As individuals ALSO YES
- Improved security of the Internet (SSL)
- Laid foundations for the future (signatures, blockchains …)
- Still challenges for use for confidentiality and privact

RSA in 2018

- Security well understood and discussed
- RSA is here to last
- Provided we keep an eye on cryptanalysis
- And on Quantum machines
- Still work on alternatives should go on
- And care should be exercised !

Oracles strike again

The ROBOT Attack

Paper | Play CTF

Test Server

Return Of Bleichenbacher's Oracle Threat

Hanno Böck, Juraj Somorovsky (Hackmanit GmbH, Ruhr-Universität Bochum), Craig Young (Tripwire VERT)

Return Of Bleichenbacher's Oracle Threat (ROBOT)

https://robotattack.org/

Hanno Böck , Juraj Somorovsky[1,2], and Craig Young[3]

[1]Ruhr-Universität Bochum
[2]Hackmanit GmbH
[3]Tripwire VERT

December 12, 2017